30th Summer Sschool and International Symposium on the Physics of Ionized Gases
August 24-28, 2020, Šabac, Serbia

Transition from electron avalanche number distributions to formative time delay distributions for multielectron initiation and streamer breakdown mechanism (I)

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1. Introduction

- Furry 1937 and Wijsman 1949 derived the electron number distribution of an avalanche by studying the fluctuation phenomena in electron avalanches **initiated by one particle**;
- the electron number distribution of an avalanche **for a large number of electrons** can be approximated by the **exponential distribution**;
- Raether 1964 and coworkers reported numerous experimental distributions of carrier numbers of electron avalanches;
- some experimental distributions show a **deviation from** the Furry and Wijsman distribution at higher values of the reduced electric field;
- for description of this deviation the Polya distribution function was introduced (Byrne 1962, Lansiart et al. 1962, Cookson et al. 1966a, Genz 1973):

$$\rho_N(n) \equiv \rho_N(n,d) = \frac{k}{\Gamma(k)\bar{n}(d)}\left[\frac{kn}{\bar{n}(d)}\right]^{k-1} e^{-\frac{kn}{\bar{n}(d)}};$$

- a review of avalanche models can be found in Alkazov 1970;
- in Jovanović et al. 2019, the experimental results for **the single electron initiation** were modeled using the Monte Carlo simulation;
- a generalization of the electron avalanche statistics for **multielectron initiation** with *fixed* and *Poisson-distributed* number of initiating electrons was proposed in Stamenković et al. 2018, Marković et al. 2019;
- in Stamenković et al. 2020, the statistics of secondary electron avalanches with **ion-induced electron emission** in air was based on NBD and its mixtures;
- the time response function of a spark counter was investigated by Devismes et al. 2002, while Mangiarotti et al. 2002 supposed the time delay and its fluctuations originate from the avalanche growth;
- analytic expressions for the shape of the time response function were derived for the **single cluster** avalanche without the space charge effect, as well as for the **multicluster** environment with the effects of space charge;
- in Gobbi et al. 2003, the fluctuation theory developed in Mangiarotti et al. 2002 was applied to measurements of spark counters and extended to other counters;
- in this paper (designated as I), **the number distributions of electron avalanches** and **formative time delay distributions** for **streamer breakdown mechanism** are studied;
- in Marković et al. 2020 (paper designated as II) the formative time delay distributions for **Townsend breakdown mechanism** are studied.

Acknowledgments: The authors are grateful to the Ministry of Education, Science, and Technological Development of the Republic of Serbia for partial financial support (contract number 451-03-68/2020-14/200124)

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2. Transition from the electron avalanche number distributions to the formative time delay distributions for streamer breakdown mechanisms and multielectron initiation

- the fluctuation in avalanche growth can be described as a fluctuation in final, critical number of electrons n_c;
- the formative time delay t_f is approximately equal to the time the avalanche takes to build up to the certain magnitude n, usually to the critical number of electrons $n_c \approx 10^8$ (Raether 1964);
- **the fluctuation in avalanche growth** can be described **as a fluctuation in avalanche length** instead of final number of electrons;
- for mathematical derivation of probability density form, the multiplication N at fixed length d is transformed into the length L at fixed multiplication n_c;
- the replacement of random variables was carried out and probability $P_N(n) = \int \rho_N(n)dn$ that the multiplication N is less than n is replaced by the probability $P_L(l) = \int \rho_L(l)dl$ that avalanche length is less than l:

$$\rho_N(n)dn = \rho_L(l)dl \qquad (1)$$

- the new probability $P(L > l)$ is introduced – probability that the avalanche will grow longer than a length l;
- on the other hand, $P(L > l)$ is equal to the probability $P(N < n_c, d = l)$ that over a length l the avalanche has not yet reached the critical number of electrons n_c:

$$P(L > l) = P(N < n_c, d = l)$$

- therefore, the probability $P_L(l)$ that avalanche length is less than l is:

$$P_L(l) = 1 - P(L > l) = 1 - P(N < n_c, d = l) = 1 - P(n_c, l) \qquad (2)$$

- $\rho_N(n)$ dependence on avalanche length d is assumed into the form:

$$\rho_N(n) \equiv \rho_N(n,d) = \frac{1}{\bar{n}(d)} f\left(\frac{n}{\bar{n}(d)}\right) \qquad (3)$$

- the new variable $x = n/\bar{n}$ is introduced;
- taking into account $\bar{n}(l) = k\exp(\alpha l)$, the probability density $\rho_L(l)$ follows:

$$\rho_L(l) = \frac{dP_L(l)}{dl} = -\frac{dP(n_c,l)}{dl} = -\frac{dP(n_c,l)}{dx}\frac{dx}{dl} = \alpha\, x(l) f[x(l)] = \frac{\alpha n_c}{\bar{n}(l)} f\left(\frac{n_c}{\bar{n}(l)}\right) \qquad (4)$$

- in the case of Polya probability density distribution for the electron number in the avalanche at fixed distance d (comparing with (3)):

$$\rho_N(n) \equiv \rho_N(n,d) = \frac{k}{\Gamma(k)\bar{n}(d)}\left[\frac{kn}{\bar{n}(d)}\right]^{k} \exp\left(-\frac{kn}{\bar{n}(d)}\right) \qquad (5)$$

- probability density $\rho_L(l)$ takes the form:

$$\rho_L(l) = \frac{\alpha k n_c}{\Gamma(k)\bar{n}(l)}\left[\frac{kn_c}{\bar{n}(l)}\right]^{k-1} \exp\left(-\frac{kn_c}{\bar{n}(l)}\right) = \frac{\alpha}{\Gamma(k)}\left[\frac{kn_c}{\bar{n}(l)}\right]^{k} \exp\left(-\frac{kn_c}{\bar{n}(l)}\right) \qquad (6)$$

- the previous relation is further transformed into **the formative time delay** probability density $\rho_T(t_f)$ by using

$$\bar{n}(l) = k\exp(\alpha l) = k\exp(\alpha w_e t_f)$$

$$\rho_T(t_f) = \frac{\alpha w_e}{\Gamma(k)}\left[\frac{n_c}{\exp(\alpha\, w_e\, t_f)}\right]^{k} \exp\left(-\frac{n_c}{\exp(\alpha\, w_e\, t_f)}\right) \qquad (7)$$

- α is the Townsend first electron ionization coefficient, w_e is the electron drift velocity and k is the number of initiating electrons;
- the formative time distributions for streamer breakdown mechanism at different number of initiating electrons k are presented in Figure 1;

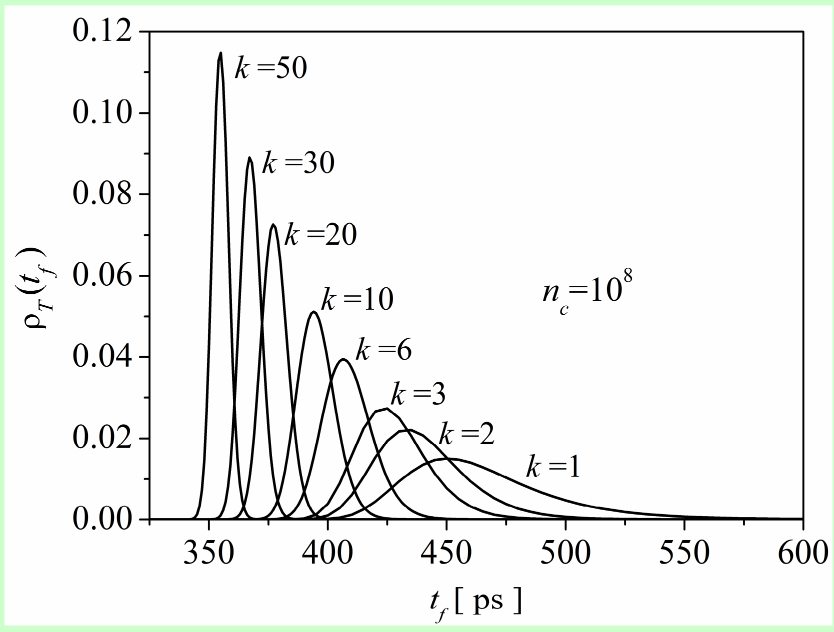

Figure 1: The formative time distributions for streamer breakdown mechanism at different number of initiating electrons k.

- when the number of initiating electrons k is small, the formative time delay distributions are asymmetric with pronounced right tail. With increasing k, the formative time distributions shift to the shorter formative times and become narrower and higher, more symmetric and Gauss-like. As statistical tests show, for $k > 10$ the hypothesis that the formative time distributions are Gaussians cannot be rejected.