

# INTERACTIONS OF IONS WITH GRAPHENE-SAPPHIRE-GRAPHENE COMPOSITE SYSTEM: STOPPING FORCE AND IMAGE FORCE

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**Abstract.** We derive general expressions for the stopping and image forces on an external charged particle moving parallel to a sandwich-like structure consisting of two doped graphene sheets separated by a layer of sapphire ( $\text{Al}_2\text{O}_3$ ) in order to study the effects of plasmon-phonon hybridization on those forces.

**Modeling of the system.** We use a Cartesian coordinate system with coordinates  $\{\vec{R}, z\}$ , where  $\vec{R} = \{x, y\}$  is a two-dimensional (2D) position vector in the  $xy$ -plane. Our system consists of two graphene sheets that are placed in the planes  $z = a/2$  and  $z = -a/2$ , as depicted in Fig. 1, with the space between them being a sapphire layer of thickness  $a$ . The graphene sheets are described by 2D response functions,  $\chi_1(q, \omega)$  and  $\chi_2(q, \omega)$ , for their non-interacting electrons and the sapphire layer is described by its local dielectric function  $\epsilon_s(\omega)$  [1]. We assume that an incident particle with charge  $Ze$  and velocity  $v$  is moving parallel to this composite structure at distance  $b$  from the top graphene surface.

In our previous publication [2] we derived an expression for the screened Coulomb interaction as:

$$W(\vec{q}, \omega, z, z') = \frac{2\pi}{q} e^{-q|z-z'|} + \frac{2\pi}{q} \left[ \frac{1}{\epsilon(\vec{q}, \omega)} - 1 \right] e^{-q(z+z'-a)},$$

where the latter part is the induced Coulomb interaction,  $W_{ind}(\vec{q}, \omega, z, z')$ , with  $\vec{q} = \{q_x, q_y\}$  being the momentum transfer vector parallel to the  $xy$ -plane and  $q = \sqrt{q_x^2 + q_y^2}$ , whereas the effective 2D dielectric function  $\epsilon(\vec{q}, \omega)$  is written as:

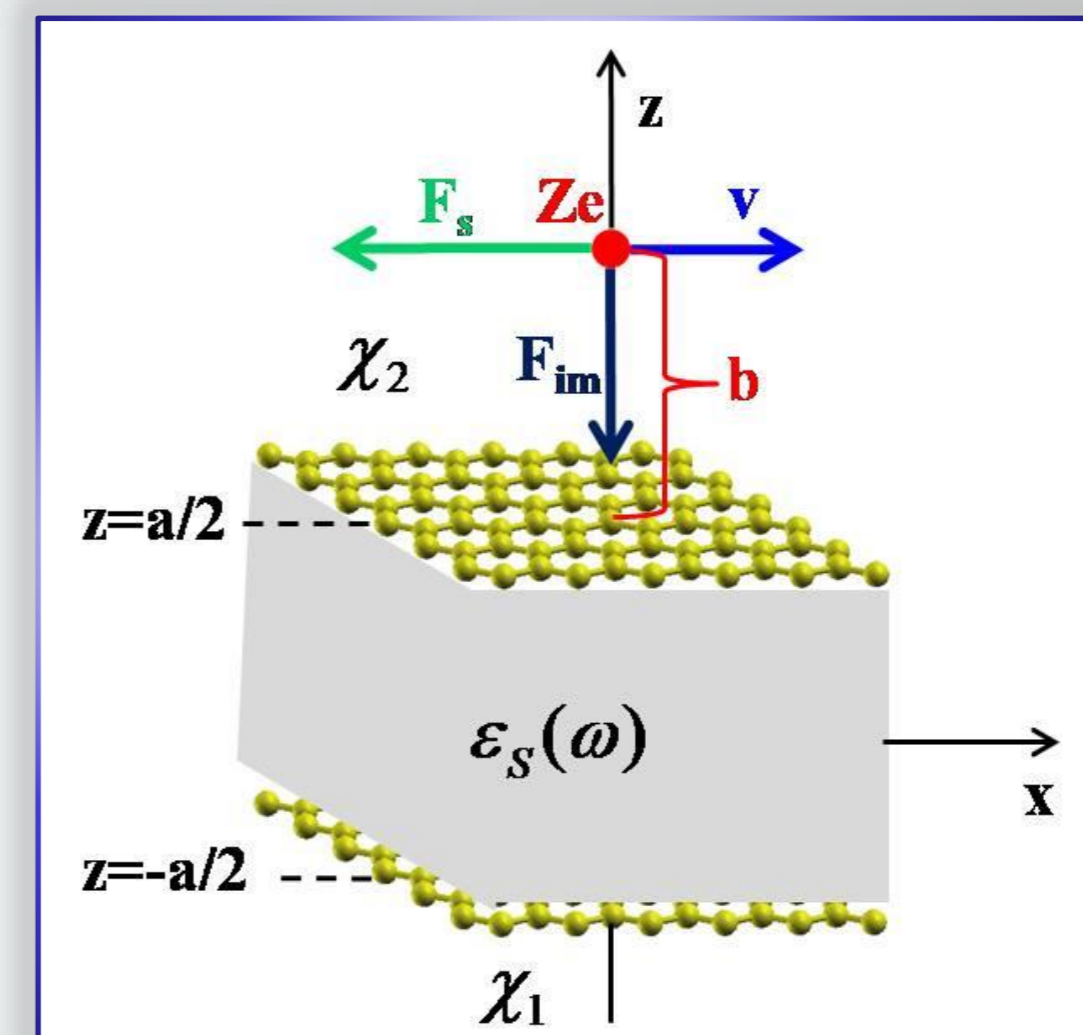
$$\epsilon(q, \omega) = \frac{1}{2} \left[ 1 + \epsilon_s(\omega) \coth(qa) + \frac{4\pi e^2}{q} \chi_2 \right] - \frac{1}{2} \frac{\epsilon_s^2(\omega) \text{cosech}^2(qa)}{1 + \epsilon_s(\omega) \coth(qa) + \frac{4\pi e^2}{q} \chi_1}.$$

**Results.** Charge density of our external point charge  $Ze$  may be written as  $\rho_{ext}(\vec{R}, z, t) = Ze \delta(\vec{R} - \vec{v}t) \delta[z - (a/2 + b)]$ . Then, the induced potential in the region above the upper graphene sheet may be expressed as:

$$\Phi_{ind}(\vec{R}, z, t) = \frac{Ze}{(2\pi)^2} \int W_{ind}(\vec{q}, \vec{q} \cdot \vec{v}, z, a/2 + b) e^{i\vec{q} \cdot (\vec{R} - \vec{v}t)} d^2\vec{q}.$$

Substituting induced Coulomb interaction into previous equation and assuming that a point charge moves along the  $x$  axis, an expression for the induced potential is obtained:

$$\Phi_{ind}(x, y, z, t) = \frac{Ze}{2\pi} \int \frac{e^{-q(z-a/2+b)}}{q} \left[ \frac{1}{\epsilon(q, q_x v)} - 1 \right] e^{i[q_x(x-vt) + q_y y]} dq_x dq_y.$$



**Fig. 1:** Diagram of the stopping force  $F_s$  and the image force  $F_{im}$  that act on the point charge  $Ze$  moving parallel to the  $x$  axis with constant speed  $v$  at a fixed distance  $b$  above the graphene-sapphire-graphene composite system.

The stopping and image forces on the moving charge  $Ze$  are defined as partial derivatives of the induced potential [3]. By using those definitions and the symmetry properties of the real and imaginary parts of the dielectric function, the final forms of the stopping and image forces, respectively, are:

$$F_s = \frac{2(Ze)^2}{\pi} \int_0^\infty \int_0^\infty \frac{q_x e^{-2qb}}{q} \text{Im} \left[ \frac{1}{\epsilon(q, q_x v)} \right] dq_x dq_y,$$

$$F_{im} = \frac{2(Ze)^2}{\pi} \int_0^\infty \int_0^\infty e^{-2qb} \text{Re} \left[ \frac{1}{\epsilon(q, q_x v)} - 1 \right] dq_x dq_y.$$

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